Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

CALCULUS.

---

110. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Find the volume removed by boring an auger hole through a right circular cone,the radius of the auger being r, the radius of the cone R, and the altitude h, and the axis of the auger intersecting axis of the cone at right angles and at a distance c from the vertex.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadélphia, Pa.

Let $R^2/h^2=a^2$, and suppose the cone to have one nappe.

Then $y^2+z^2=a^2x^2$ is the equation to the cone, and $(x-c)^2+y^2=r^2$ is the equation to the auger.

When $y=ax$, $z=0$, $x_1=\dfrac{c+\sqrt{[r^2(a^2+1)-a^2c^2]}}{a^2+1}$, $x_2=\dfrac{c-\sqrt{[r^2(a^2+1)-a^2c^2]}}{a^2+1}$

Let $\sqrt{[r^2-(x-c)^2]}=y'$, $[r^2-(x-c)^2][a^2x^2-r^2+(x-c)]=A^2$.

I. When $c<r$.

$$V=4\int_{x_1}^{c+r}\int_0^{y'}\sqrt{[a^2x^2-y^2]}dxdy+4\int_0^{x_1}\int_0^{ax}\sqrt{[a^2x^2-y^2]}dxdy$$

$$=2\int_{x_1}^{r+c}\left(A+a^2x^2\sin^{-1}\frac{y'}{ax}\right)dx+\pi a^2\int_0^{x_1}x^2dx$$

$$=2\int_{x_1}^{r+c}\left(A+\frac{a^2x^2(cx+r^2-c^2)}{3A}\right)dx+\tfrac{1}{3}\pi a^2x_1^3=2\int_{x_1}^{r+c}Bdx+\tfrac{1}{3}\pi a^2x_1^3.$$

II. $c>r$, but $r<ac/(a^2+1)$.

$$V=4\int_{c-r}^{c+r}\int_0^{y'}\sqrt{[a^2x^2-y^2]}dxdy=2\int_{c-r}^{c+r}Bdx.$$

III. $c>r$, but $r>ac/(a^2+1)$.

$$V=4\int_{x_1}^{c+r}\int_0^{y'}\sqrt{[a^2x^2-y^2]}dxdy+4\int_{c-r}^{x_2}\int_0^{y'}\sqrt{[a^2x^2-y^2]}dxdy$$

$$+4\int_{x_2}^{x_1}\int_0^{ax}\sqrt{[a^2x^2-y^2]}dxdy=2\int_{x_1}^{c+r}Bdx+2\int_{c-r}^{x_2}Bdx+\tfrac{1}{3}\pi a^2(x_1^3-x_2^3).$$

Let $u=x-c$ and $l^2=R^2+h^2$.

$$\therefore A=\frac{1}{hl}\sqrt{\{(r^2-u^2)[(l^2u+R^2c)^2+h^2(R^2c^2-l^2r^2)]\}}.$$

Let $h^2(R^2c^2-l^2r^2)=b^2l^4$, $R^2c/l^2=d$.

Then $A=(l/h)\sqrt{\{(r^2-u^2)[b^2+(u+d)^2]\}}$.

This can be integrated by a process similar to that employed by Professor Finkel in Vol. V, No. 1, pages 20, 21.

$$\text{Let } Rc-lr, \text{ then } A=\frac{1}{h}(lu+Rr)\sqrt{(r^2-u^2)}=\frac{R}{rh}(cu+r^2)\sqrt{(r^2-u^2)}.$$

$$\therefore\ V=\frac{2R}{hr}\int_{-r}^{r}(cu+r^2)\sqrt{(r^2-u^2)}du+\frac{2Rr}{3h}\int_{-r}^{r}\frac{(u+c)^2du}{\sqrt{(r^2-u^2)}}$$

$$=\frac{2\pi Rr}{3h}(2r^2+c^2).$$

111. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

(*a*). Find the dimensions of a cup, capacity c, in the form of a frustum of a regular pyramid of n faces, so that its internal surface is a minimum.

(*b*). Find the dimensions of a cup, capacity c, in the form of a frustum of a hyperbolioid or of a paraboloid, whichever it is, so that its internal surface is a minimum.

Solution by the PROPOSER.

(*a*). Let $PO=x$, $PG=y$, $\angle AOB=\angle DGE=2\pi/n$, $AO=r$, where O is the center of the circle circumscribing the larger base and G the center of the circle circumscribing the smaller base.

Then $DG=ry/x$, $AB=2r\sin(\pi/n)$, $DE=(2ry/x)\sin(\pi/n)$. Area $DGE=(r^2y^2/x^2)\sin(\pi/n)\cos(\pi/n)$.

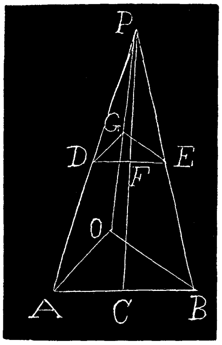
$\therefore$ Area of upper base$=(nr^2y^2/x^2\sin(\pi/n)\cos(\pi/n)$.

$PC=\sqrt{(PA^2-AC^2)}=\sqrt{[x^2+r^2-r^2\sin^2(\pi/n)}$

$=\sqrt{[x^2+r^2\cos^2(\pi/n)}$.

Similarly $DF=(y/x)\sqrt{[x^2+r^2\cos^2(\pi/n)]}$.

Area $ADEB=r\sin(\pi/n)\sqrt{[x^2+r^2\cos^2(\pi/n)]}[(x^2-y^2)/x^2]$.



The surface is the least when the upper or smaller base is the bottom of the cup.

Total surface of cup$=u$, volume$=c$.

$$\therefore\ u=nr\sin(\pi/n)\sqrt{[x^2+r^2\cos^2(\pi/n)]}\left(\frac{x^2-y^2}{x^2}\right)+\frac{nr^2y^2}{x^2}\sin(\pi/n)\cos(\pi/n)$$

$c=\frac{1}{3}nr^2\sin(\pi/n)\cos(\pi/n)\left(\dfrac{x^3-y^3}{x^2}\right)$. Let $(r/x)\cos(\pi/n)=\tan\theta$.

$\therefore u=n\tan\theta\sec\theta\sec(\pi/n)\sin(\pi/n)(x^2-y^2)+ny^2\tan^2\theta\sec(\pi/n)\sin(\pi/n).(1)$.

$c=\frac{1}{3}n\tan^2\theta\sec(\pi/n)\sin(\pi/n)(x^3-y^3)\ldots.(2)$.